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ANALYSIS OF THE RAYLEIGH NUMBER IN THE AREA OF STEEL RECTANGULAR SECTIONS IN THE CONDITIONS OF STEDY AND UNSTEDY HEAT FLOW

Rafał WYCZÓŁKOWSKI[1]
Czestochowa University of Technology
Department of Industrial Furnaces and Environmental Protection, Poland

Abstract

The paper presents the experimental measurements dedicated to the research for the Rayleigh number (*Ra*) in the area of rectangular steel sections. This problem is associated with the analysis of the free convection which takes place in the heating of sections bundles during the heat treatment. The study was conducted for both steady and transient heat transfer. The values of the *Ra* number obtained for the tested sections allow to describe the phenomenon of convection on the basis of a very simple criterial dependence. It greatly simplifies the mathematical description of the heat transfer phenomenon in the concerned charges.

Keywords: Rayleigh number, free convection, steel section, heat treatment

1. INTRODUCTION

In the heat treatment processes, steel elements which belong to a range of long products can be heated in the form of bundles [7]. Bundles formed from different types of long elements are shown in Fig. 1. One of the kinds of these charges are bundles formed from rectangular sections (Fig. 1d).

[1] Corresponding author: Department of Industrial Furnaces and Environmental Protection, Czestochowa University of Technology, Al Armii Krajowej 19, 42-200 Częstochowa, Poland, e-mail: rwyczolkowski@wip.pcz.pl, tel. +48662680780

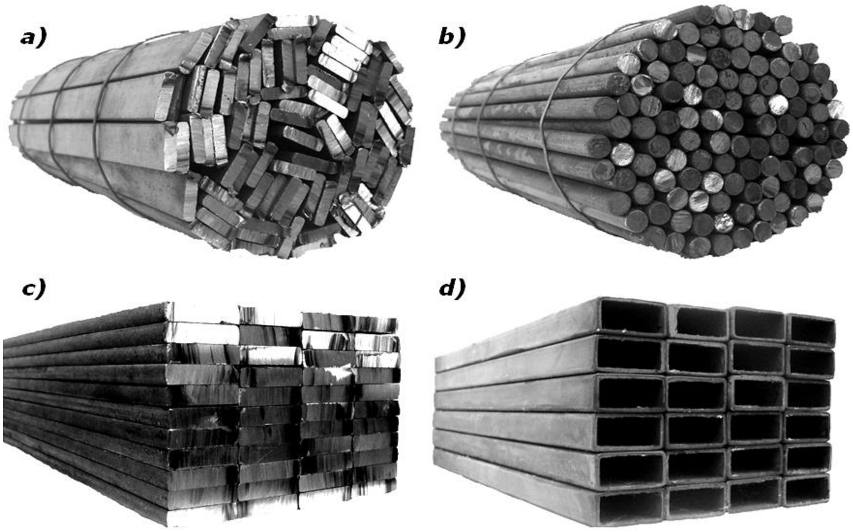

Fig. 1. Examples of a bundles formed from different types of long elements: a) c) flat bars, b) round bars, d) rectangular sections

To provide a necessary quality of the heat treated product and to minimize the production cost it is necessary to optimize the heating process. To achieve this optimization we need to establish two parameters: heating curve and heating time. These parameters can be obtained by numerical calculations. Making such calculations requires information about the thermal properties of the treated charge. As far as a porous charge is concerned the main thermal property is the effective thermal conductivity k_{ef}. This quantity is calculated with the help of a mathematical model which describes a complex heat flow that takes place in the area of the considered charge. This approach uses thermal analogy and the concept of elementary cell [11, 9].

Due to a large portion of hollow spaces the bundles of steel rectangular sections are characterized by porosity even exceeding 85%. For this reason one of the factors influencing the heating process of these elements is free convection which occurs in the inner regions of the sections. Thus, the mathematical model, which is used to calculate the effective thermal conductivity of these charges, must take into consideration a mathematical description of this phenomenon. In some cases description of the convection can be done by simple criterial dependences which arises from dimensional analysis and similarity theory. The criterial quantity of these dependences is a Rayleigh number (*Ra*), which expresses the intensity of a free convection. For this reason the knowledge about the values which take *Ra* parameter during the heating of the sections bundles is an essential problem.

2. MATHEMATICAL DESCRIPTION OF A FREE CONVECTION

A phenomenon of a free convection is a mixture of fluid flow and heat transfer. For this reason a system of differential equations with suitable conditions of uniqueness is used to describe this phenomenon. The differential equations which are taken into account are: equation of continuity, equation of fluid motion and equation of energy balance [6, 1]. The conditions of uniqueness along with mentioned differential equations create a general mathematical description of a free convection phenomenon. The analytical solutions of this problem exist only for few simple cases.

Due to mentioned difficulties, the adequately elaborate results of experimental studies are of the main significance in the analysis of free convection. In this case the main working method is the similarity theory and dimensional analysis. As it was already mentioned, a measure of the intensity of natural convection is the Rayleigh number [1, 2]. This parameter is defined as the product of the Graschhoff number *Gr* which describes the relationship between buoyancy and viscosity within a fluid, and the Prandtl number *Pr* which describes the relationship between momentum diffusivity and thermal diffusivity. For the enclosed space, which is made up by the spaces within the sections, the *Ra* number is described by the following equation:

$$Ra = Gr \cdot \mathrm{Pr} = \frac{g \cdot \beta \cdot \Delta t \cdot d_h^3}{\nu^2} \cdot \mathrm{Pr}\,. \tag{2.1}$$

Here d_h is inside hydraulic diameter of the section; Δt is temperature difference between the surface of the section perpendicular to the direction of heat flow, K, g is acceleration due to gravity, m/s^2; ν is kinematic viscosity, m^2/s; β is thermal expansion coefficient, K^{-1}.

For the flow of heat in confined spaces, when the *Ra* number is contained in the range of $1{,}7\times10^3 \div 10^{10}$, the heat flow intensification caused by the occurrence of natural convection can be expressed as the enlargement of heat conduction in a fluid [7]. For this purpose, the equivalent fluid heat conductivity, e_eq, is used:

$$k_{eq} = e_{eq} \cdot k_f \tag{2.2}$$

where k_f denotes the thermal conductivity of the fluid. Coefficient e_{eq}, expressing the intensification of heat transfer compared to the stationary fluid, is determined from the power relationship:

$$e_{eq} = C \cdot Ra^A \tag{2.3}$$

Constants C and A from eq. (2.3) depend on the value of Ra number. On the basis of the experimental test results, the following values were determined [4]:

$$C = 0{,}105 \quad A = 0{,}3 \quad \text{for} \quad 1{,}7\times10^3 < Ra < 10^6 \tag{2.4}$$

$$C = 0{,}4 \quad A = 0{,}2 \quad \text{for} \quad 10^6 < Ra < 10^{10} \tag{2.5}$$

For the whole range of the Ra number value the following approximation can be also applied:

$$C = 0{,}18 \quad A = 0{,}25 \quad \text{for} \quad 1{,}7\times10^3 < Ra < 10^{10} \tag{2.6}$$

3. TESTS IN THE CONDITIONS OF STEDY HEAT FLOW

To determine the Rayleigh number in the area of the sections bundles for the steady heat transfer conditions a heating chamber of a stand has been used to measure the effective thermal conductivity of porous charges. The scheme of construction of this chamber is shown on Fig. 2a. The measurement space form a cuboid steel retort with dimensions of basis 400×400 mm and height 200 mm, in which the investigated samples are placed as shown in Fig. 2.

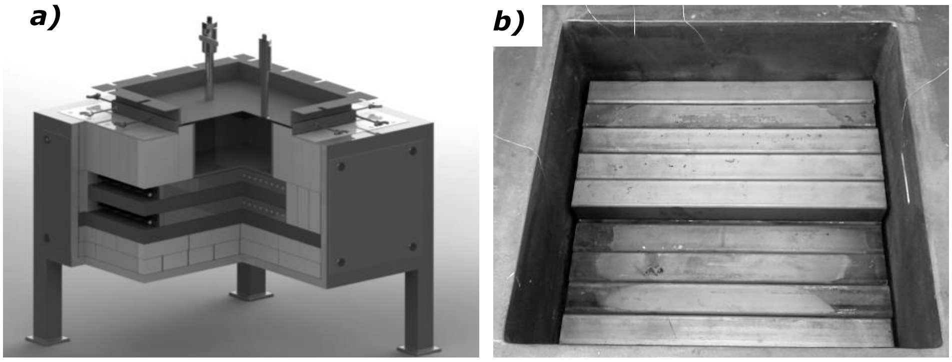

Fig. 2. a) A view of the structure of the heating chamber of the experimental stand, b) One of the samples located inside the retort

Under the retort a set of two electrical heaters is located. The operation of these heaters allows to get a one-dimensional heat flow in the area of the heated samples. At the top the retort is closed by a steel cover.
The idea of performed investigations is illustrated on Fig. 3a. They were connected with heating of the investigated samples by one-dimensional constant heat flux q. This process was carried out till the steady state was obtained. When this state was reached, the temperature was measured within the selected section. The measurement points were located on the opposite, horizontal walls of particular section. Knowing the dimensions of the section and the temperature difference along its depth, the value of the *Ra* number can be calculated in accordance with equation (2.1). Temperature measurements at the sections surface were made by using *K*-type jacket thermocouples. These sensors ware connected to a microprocessor thermometer EMT 200 [8]. This set allows to measure the temperature with accuracy of 0,1°C. The measurement tips of the thermocouples were mounted at the sections surfaces in the small furrows. For that purpose a heat-resistant silicone was used. The way of the thermocouples mounting at the section surface is illustrated on Fig. 3b.

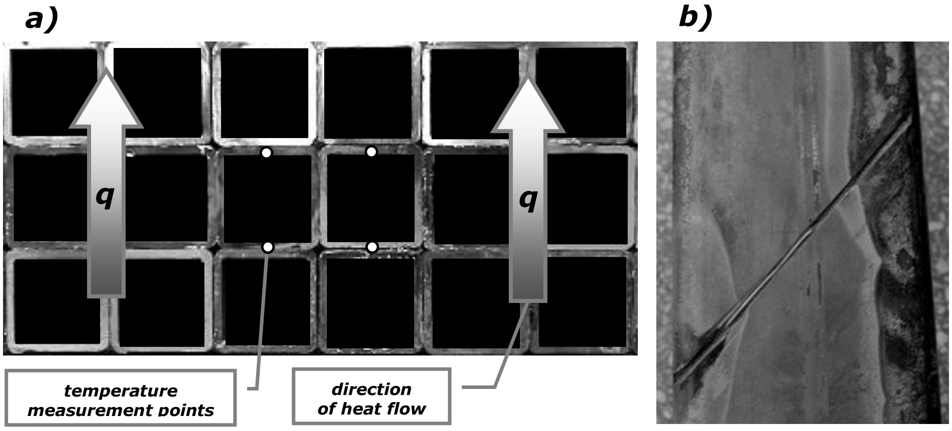

Fig. 3. a) A drawing illustrating the idea of the measurements dedicated to research for the *Ra* number. b) A manner of the thermocouple fix to the section's surface.

During the tests, it was essential to establish the values of the *Ra* number in the range of temperature as wide as possible. Hence the investigated samples were heated for gradually increasing values of the power supplied to the heater. This was feasible due to the autotransformer which was fitted to the experimental stand. When for a given power a steady state was obtained the temperatures were measured. Then the supplied power to the heater was increased and the whole procedure was repeated. The measurements were made for ten different values of supplied power *N*. This parameter was changed in the range of 200÷3000v. For this reason, the values of the *Ra* number for investigated sections were established for temperature in the range of 70÷650°C.

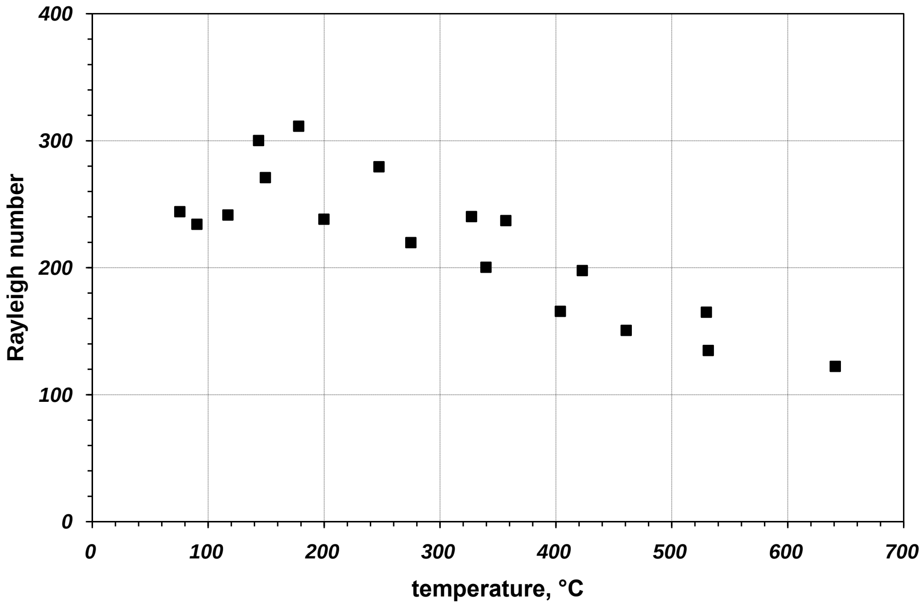

Fig. 4. The values of the *Ra* number for the 40×20 mm sections obtained in the steady conditions

The tested samples were built from three types of sections, which differed by transverse dimensions and wall thickness and had form of flat packed beds. Dimensions of these sections were as follows: 40×20 mm (wall thickness was 2 mm), 60×60 mm and 80×80 mm (wall thicknesses in the last two cases were 3 mm). The values of the Rayleigh number obtained for sections 40×20 mm are shown on Fig. 4 The temperature which corresponds to the given value of the *Ra* number was calculated as an average of the temperatures measured on the walls of the considered section. It was noted that the maximum values for the investigated parameter were in the temperature range of 100÷200°C, for which the maximum value of the *Ra* number was equal to about 300. For the temperatures above 200°C the values of the analyzed parameter decreased linearly. In temperature of 650°C the value of *Ra* number was only 120.
As it is known, the critical value of the *Ra* number is 1 700 [10, 5] and after exceeding it the fluid-filled space starts reaching the conditions of convective movements. Therefore in the inner area of the 40×20 mm sections in the

conditions of the steady heat flow independent from the temperature, a free convection of gas does not occur.

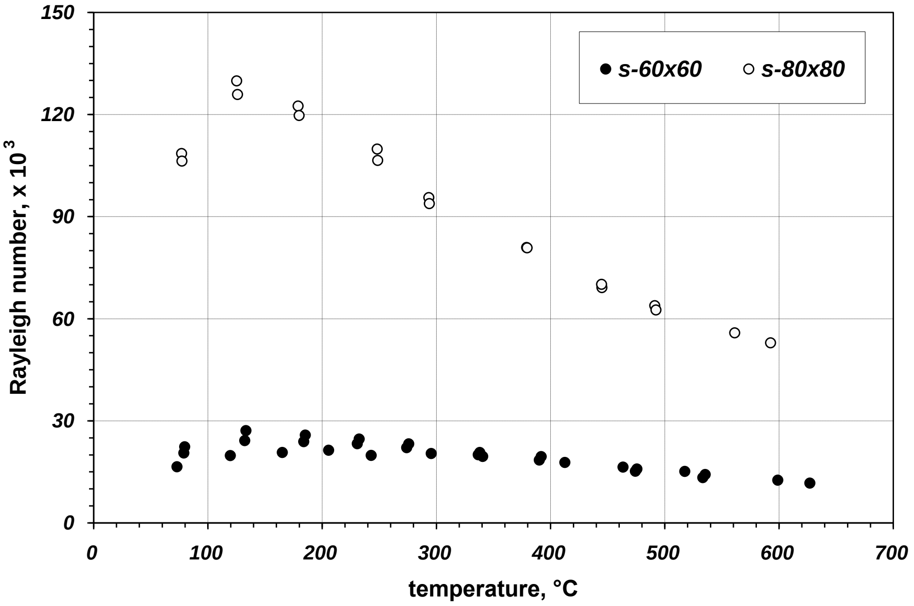

Fig. 5. The values of the *Ra* number for the 60×60 mm and the 80×80 mm sections obtained in the steady conditions

Fig 5 shows the values of Rayleigh number obtained for 60×60 mm and 80×80 mm sections. Similarly to the previous case the maximal values of the *Ra* parameter are presented in the temperature range of 100÷200°C. The values obtained for the 60×60 mm sections are 27 000, and for the 80×80 mm sections are 130 000. As it can be noted, the values of the *Ra* number increase drastically with the rise of the dimensions of the area in which the heat flow occurs. This effect results from the fact that the Rayleigh number depends on the cube of the dimensions of analyzed region. In higher temperatures the values of an analyzed parameter decreases linearly. It can be seen from the obtained results that the increase of 25% in the area of heat flow gives rise to the Rayleigha number by about 5 times. Therefore, a relatively small increase in the area of heat flow in fluid results in great intensification of free convection. In this case, the critical value of the *Ra* number has been exceeded in the whole range of temperature. Based on that, we can say that free convection in the heating of 60×60 mm sections as well as of the 80×80 mm sections is presented in the entire process.

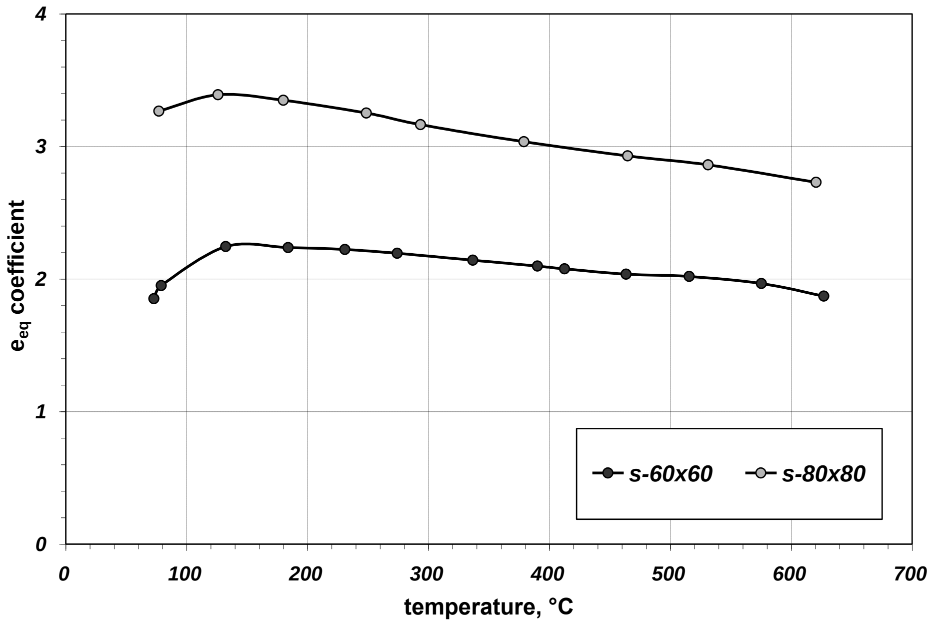

Fig. 6. Results of the calculations of coefficient e_{eq} for the 60×60mm and the 80×80 mm sections for the steady conditions

An equation (2.3) has been used to establish how the free convection affects the intensification of heat flow in the inner space of the 60×60 and 80×80 sections. The measure of this intensification is the values of the e_{eq} coefficient. Due to the value of the *Ra* number, in the calculations of the coefficient e_{eq} one should apply constants *C* and *A*, given in relation (2.4). The results of these calculations are shown in Fig. 6. For the 60×60 mm sections an average value of coefficient e_{eq} is around 2 and for the 80×80 mm sections it is around 3,5. In both cases for temperatures higher than 150°C the value of c_{eq} slightly decreases. This decrease has almost linear character.

4. TESTS IN THE CONDITIONS OF UNSTEDY HEAT FLOW

The studies of the *Ra* number in the unsteady conditions were carried out in the same manner as for the steady conditions. The measurements of the temperature was made in the area of the sections which were heated in the electric furnace. For this purpose a laboratory chamber unit was used (Fig. 7a). In this case the tested samples were bundles of the sections. Fig 7b shows one of the bundles within the furnace.

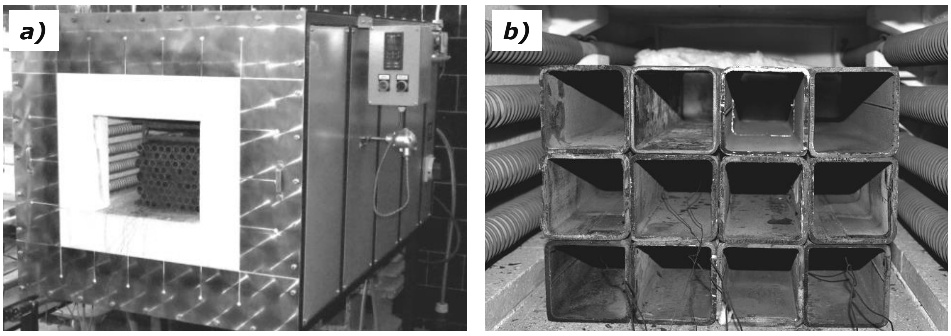

Fig. 7. a) The chamber furnace applied to the tests, b) One of the bundles during the tests in the furnace

During the experiment the furnace temperature was changed from room value to 750°C. At this process the measurements of the temperature in the area of the sections were made every 10 mins. The tests were carried out for the bundles of the 60×60 mm and the 80×80 mm sections. The results of the *Ra* number in this experiment are illustrated on Fig. 8.

The qualitative changes of the *Ra* number versus temperature for the tested sections in the unsteady conditions are very similar to the results obtained in the steady conditions. However, when the value of the *Ra* parameter is concerned there is a great difference between these two cases. For the unsteady conditions the values of the *Ra* number are higher by ten times in comparison with the steady state. For the 60×60 mm sections the *Ra* number had a maximum value in 85°C which was 140 000. For the final heating temperature (700°C) this criterion has fallen to 1 160. For the 80×80 mm sections the *Ra* number had a maximum value in 80°C which was up to 1 150 000. At the end of the heating process the value of this criterion decreased to 18 500.

Obtained results show that the free convection in the same geometrical systems under transient conditions in relation to steady once is much more intense phenomenon. This is due to much larger temperature differences that occur when transient heat flow takes place. Particularly large temperature differences within the sections occur with such dynamic processes as heating during heat treatment.

Fig. 9 presents the calculation results of coefficient e_{eq} performed for the *Ra* parameter values from Fig 8. For the 60×60 mm sections occurrence of transient convection in comparison with stagnant gas, depending on the temperature, increases heat transfer rates in the range of 3,5 to 1,5. The observed decrease in the value of the coefficient e_{eq} as a function of temperature is almost linear in nature. The same relationship has been observed for sections 80×80 mm. In this case, depending on the temperature this coefficient e_{eq} has a value from 6 to 2,5.

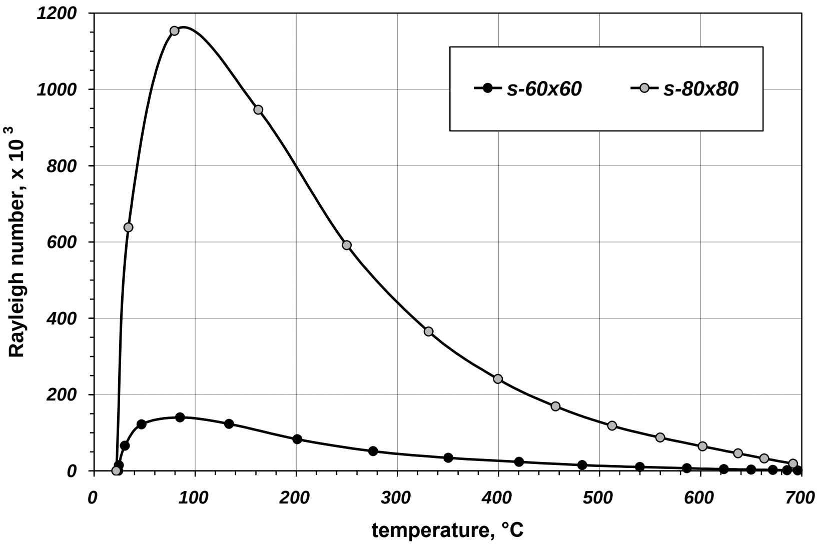

Fig. 8. The values of the *Ra* number for the 60×60 mm and the 80×80 mm sections obtained in the transient conditions

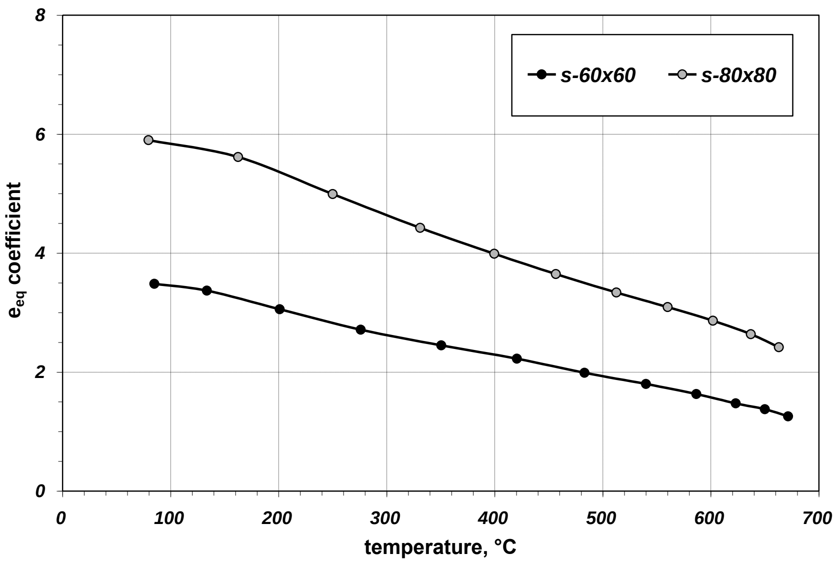

Fig. 9. Results of the calculations of coefficient e_{eq} for the 60×60mm and the 80×80 mm sections for the unsteady conditions

5. SUMMARY

This paper presents experimental measurements dedicated to research of the Rayleigh number. This investigations are related to the process of heat flow in the area of the bundles of steel rectangular sections. It was found that for transient heat flow the values of the *Ra* number are of the order of magnitude higher as compared to the flow in steady conditions. The values of *Ra* number obtained for the studied sections are in the range for which the effect of convection can be expressed using a very simple criterion dependence. This approach greatly simplifies the mathematical description of the phenomenon of heat flow in the concerned charges.

The final outcome of the conducted studies will be the developed universal analytical model for the determination of the effective thermal conductivity of bundles of various types of long elements. This model will be implemented as a numerical procedure in thermal modules designed to the modelling of heat treatment processes. Its operation will enable more precise evaluation of the field of temperature within the space of either heated or cooled sections bundles.

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ANALIZA LICZBY RAYLEIGHA W OBSZARZE STALOWYCH PROFILI PROSTOKĄTNYCH W WARUNKACH USTALONEGO I NIEUSTALONEGO PRZEPŁYWU CIEPŁA

S t r e s z c z e n i e

W artykule przedstawiono wyniki badań eksperymentalnych, których celem było określonie liczby Rayleigha (Ra) dla profili prostokątnch, nagrzewanych w warunkach ustalonej i nieustalonej wymiany ciepła. Analizie poddano trzy rodzaje profili: 40×20 mm, 60×60 mm oraz 80×80 mm. Badania dla warunków ustalonych przeprowadzono w komorze grzewczej stanowiska do pomiarów efektywnej przewodności cieplnej wsadów porowatych. Natomiast badania w warunkach nieustalonych przeprowadzono w elektrycznym piecu komorowym. Charakter zmian liczby Ra w funkcji temperatury, dla obu sposobów nagrzewania jest podobny. W każdym z przypadków, maksymalną wartość parametr Ra uzyskuje w przedziale temperatury około 100÷200°C. Dla profili 80×80 mm parametr ten jest około sześciokrotnie większy w porównaniu z profilami 60×60 mm. W przypadku nieustalonego przepływu ciepła, bezwzględne wartości parametru Ra są o rząd wielkości większe od wartości uzyskanych dla warunków ustalonych. Odnotowane liczby Ra nie przekroczyły wartości 10^7. Maksymalna wartość tego parametru wyniosła około $1{,}2\times10^6$. Zatem przy nagrzewaniu analizowanych profili, występującą w ich wnętrzu konwekcję, można traktować jako intensyfikację przewodzenia w wypełniającym je powietrzu. Wyniki przedstawionych badań posłużą do analizy wpływu występującej wewnątrz profili konwekcji, na proces nagrzewania wiązek tych elementów. Ostatecznym wynikiem prowadzonych w tym zakresie analiz, będzie opracowanie modelu do wyznaczania efektywnej przewodności cieplnej tego wsadu.

Słowa kluczowe: liczba Rayleigha, konwekcja swobodna, profile stalowe, obróbka cieplna

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