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DAMAGE IDENTIFICATION OF A STEEL PLATE USING VIBRATION METHODS

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Abstract

This paper presents an experimental study on the detection and localization of damage in a steel plate using vibration-based methods. Dynamic impulse tests were conducted on both undamaged and damaged plate to determine changes in modal parameters, such as eigenfrequencies and mode shapes. Several damage detection methods, including the Modal Assurance Criterion (MAC), Coordinate Modal Assurance Criterion (COMAC), Mode Shape Curvature (MSC), Mode Shape Curvature Square (MSCS), and Damage Index (DI), were applied. The results showed that while MAC and COMAC effectively detected damage, they did not provide information about the location. MSC and MSCS indices demonstrated improved damage localization, with the MSCS being particularly effective. The DI index provided the clearest and most accurate representation of the damage location. These findings highlight the potential of these methods for non-destructive testing and structural health monitoring.

Keywords: vibration-based damage identification method, non-destructive testing, structural health monitoring, impulse test, steel plate

1. INTRODUCTION

Ensuring the safety of users during building operations is a priority for designers, contractors, and institutions involved in maintenance and quality control. To prevent failures, regular monitoring of the technical condition of the structure or its elements is conducted, and if necessary, renovation and repairs are carried out. These activities directly contribute to user safety. However, when damage occurs, it is crucial to identify the failure location as quickly as possible.

Today, Non-Destructive Testing (NDT), including methods based on the dynamic characteristics of structures, has gained popularity (Verma et al. 2013; Zielińska and Rucka 2018). Due to the fact that dynamic parameters are inseparable from the structure itself, any damage results in a change in these parameters. Therefore, determining the dynamic characteristics of both undamaged and damaged

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structures and comparing the obtained data enables the identification of damage. Vibration-Based Damage Identification Methods (VBA DIMs) allow for both damage detection and location identification (Avci et al. 2021; Das et al. 2016; Magalhães et al. 2012; Magalhães 2010).

When monitoring the technical condition of a structure is necessary, Structural Health Monitoring (SHM) is carried out, allowing for a quick reaction in the event of damage. SHM has a great social benefit, in particular due to the monitoring of economically or politically important facilities, where the protection of human life and health is particularly important. SHM is also important for the preservation of architectural heritage, enabling the identification and intervention in the event of damage to historic buildings (Pallarés et al. 2021; Ramos et al. 2010).

Over the past few decades, algorithms have been developed for dynamic system identification, allowing for the determination of modal parameters such as eigenfrequencies, mode shapes, and damping coefficients (Maia et al. 1997; Reynders 2012). Advances in analog-to-digital transducers have made it possible to measure the structural response accurately, which has greatly contributed to the development of experimental modal identification techniques.

There are two main branches of modal analysis: Experimental Modal Analysis (EMA), which is used in this study, and Operational Modal Analysis (OMA). In EMA, both excitation and response are measured to deterministically identify modal parameters, while in OMA, only the response to naturally occurring excitations such as wind or traffic is recorded.

In building structures, excitation is typically achieved using mechanisms selected based on the size of the object being tested (Ewins 2000a). In experimental modal analysis, the impulse test and sweep test are common methods (Rucka and Wilde 2012; Schwarz and Richardson 1999). In this study, the impulse test was performed, where a modal hammer generated a short-term pulse of force, and the response was measured using accelerometers. The force was applied at various locations, while the accelerometers recorded acceleration at fixed points.

This paper aims to present a method for diagnosing and locating damage in a plate structure using vibration methods, with an experimental model of a steel plate as a case study. Dynamic impulse tests were conducted to determine the structure's dynamic characteristics. Tests were performed on both undamaged and damaged objects, allowing the use of selected indicators to detect and locate damage.

This topic has been discussed in the literature (Avci et al. 2021; Bru et al. 2023; Masciotta et al. 2017), but there are not many examples of applications.

2. DAMAGE INDICATORS BASED ON VIBRATION RESPONSE

The first method of failure detection presented here is the Modal Assurances Criterion (MAC). According to this method it is possible to diagnose damage without specifying its location, based on a comparison of the eigenvectors of the structure in two states, as described by formula (2.1):

$$MAC_{mn} = \frac{\left| \sum_{i=1}^{N_i} \left(\varphi_{i,m}^{u} \varphi_{i,n}^{d} \right) \right|^2}{\sum_{i=1}^{N_i} \left(\varphi_{i,m}^{u} \right)^2 \sum_{i=1}^{N_i} \left(\varphi_{i,n}^{d} \right)^2} \tag{2.1}$$

where $\varphi_{i,m}^{u}$ and $\varphi_{i,n}^{d}$ are the values at the i-th point (node) of the m-th and n-th eigenmode of the studied structure in the first/reference state and in the second/current state, respectively. The reference state can be the state of an undamaged object (index u – undamage), while the current state can represent a damaged object (index d – damage).

The degree of the MAC matrix is equal to the number of mode shapes taken for analysis. The degree of the MAC matrix corresponds to the number of mode shapes analyzed. Matrix elements range from 0 to 1, indicating the correlation between the structure's mode shapes in two different states (undamaged vs. damaged). A value of 0 shows no correlation, while 1 indicates complete correlation. Initially, the MAC criterion was used in the late 1970s for testing vibration orthogonality (Allemang 2003). In his work, Allemang also presented several other possible applications of this criterion, such as the validation of experimental models or the correlation of numerical models with real structures. For the purpose of detecting damage, the MAC criterion was first used in 1984 by W. M. West to examine the structural elements of a spacecraft (Doebling et al. 1998; West 1986).

An extension of the MAC criterion is the Coordinate Modal Assurance Criterion (COMAC), which allows the location of damage on the basis of comparison of eigenvectors N_m from two states counted in the nodes of the structure (2.2):

$$COMAC_i = \frac{\left|\sum_{m=1}^{N_m}\left(\varphi_{i,m}^{u}\varphi_{i,m}^{d}\right)\right|^2}{\sum_{m=1}^{N_m}\left(\varphi_{i,m}^{u}\right)^2\sum_{m=1}^{N_m}\left(\varphi_{i,n}^{d}\right)^2} \tag{2.2}$$

where $\varphi_{i,m}^{u}$ and $\varphi_{i,n}^{d}$ are the values at the i-th point (node) of the m-th form of mode shape of the structure in the reference state (u – undamage) and in the current state (d – damage) respectively.

According to the above formula, COMAC values refer to the nodes of the structure. Considering a selected number of identified pairs of corresponding eigenvectors from two states: undamaged and damaged, more detailed information about the correlation between the modes at each node of the structure is obtained (Lieven and Ewins 1988). The normalized COMAC parameter takes values between 0 and 1 for each node, where the lower the value, the greater the discrepancy between the corresponding sets of eigenmodes from the two states at a given location. Attention should be paid to areas with a high amplitude of eigenmodes, e.g. free ends of beams, where more noticeable inaccuracies of these parts of the structures may occur, which may result in a lower value of the coefficient (Ewins 2000b; Ewins 2000a).

An alternative to using the shapes of the eigenmodes of the structure for the purpose of detecting damage is to use information about the curvature carried by the derivatives of these modes. In 1991, a publication was published (Pandey et al. 1991) in which, on the basis of simple, linear models, the authors showed that the change in curvature was correlated with the location of the structure damage and the greater the damage, the greater the change in curvature. In addition, the authors showed a higher sensitivity of this method compared to the COMAC method, especially in the initial phase of damage. On this basis, the MSC (Mode Shape Curvature) index was introduced, which enables localization of damage to the structure based on changes in the curvatures of the eigenmodes from two states - before and after damage. The damage location index of the two-dimensional structure for selected mode shapes N_m should be calculated from the following formula (2.3):

$$MSC_{ij}^{2D} = \sum_{m=1}^{N_m}\left|\varphi_{ij,m}^{d}{''} - \varphi_{ij,m}^{u}{''}\right| \tag{2.3}$$

where $\varphi_{ij,m}^{u}{''}$ and $\varphi_{ij,m}^{d}{''}$ is the curvature value of the m-th eigenmode at the point with coordinates (i, j) of the tested structure in the undamaged state (u) and in the damaged state (d).

Ho and Ewins (Ho and Ewins 2000) proposed a related criterion, also based on the curvature of the mode shapes, defined as the Mode Shape Curvature Square (MSCS) damage index, presented as follows:

$$MSCS_{ij}^{2D} = \sum_{m=1}^{N_m} \left| \left(\varphi_{ij,m}^{d}{}'' \right)^2 - \left(\varphi_{ij,m}^{u}{}'' \right)^2 \right| \tag{2.4}$$

where $\varphi_{ij,m}^{u}{}''$ and $\varphi_{ij,m}^{d}{}''$ are the values of the curvature of the m-th eigenmode at the point defined by the coordinates (i, j) of the tested structure in the undamaged (u) and in the damaged state (d).

The authors (Ho and Ewins 2000) indicated that the above methods of damage detection and localization based on the eigenmodes and their derivatives are reliable. They paid special attention to the quality of the measurements performed and the areas of nodes and edges of structures where false damage may be recorded. The validity of using the curvatures of the mode shapes of natural vibrations in damage localization was also confirmed by Maeck and De Roeck in their publications (Maeck and De Roeck 2003; Maeck and De Roeck 1999).

The last method to be discussed here is the method of calculating the Damage Index (DI) proposed by Stubbs and others (Stubbs et al. 1995; Stubbs et al. 1992). This method, similarly to MSC and MSCS, uses the curvature characteristics of the structure's mode shape as the main variable in the derived damage location algorithm, based on the relative differences in the modal strain energy before and after damage. The approach presented there is described in detail using the example of beams and can be extended to plates or any three-dimensional structures. A description, application and development of this method on the example of a timber beam model can also be found in the work (Choi et al. 2008).

Considering one value of the curvature of the eigenmode for individual structure fragments (i, j) the damage index can be presented in a simpler form:

$$DI_{ij}^{2D} = \sum_{m=1}^{N_m} \left[\frac{\left(\varphi_{ij,m}^{d}{}'' \right)^2 + \sum_{j=1}^{N_j} \sum_{i=1}^{N_i} \left(\varphi_{ij,m}^{d}{}'' \right)^2}{\left(\varphi_{ij,m}^{u}{}'' \right)^2 + \sum_{j=1}^{N_j} \sum_{i=1}^{N_i} \left(\varphi_{ij,m}^{u}{}'' \right)^2} \right] \cdot \frac{\sum_{j=1}^{N_j} \sum_{i=1}^{N_i} \left(\varphi_{ij,m}^{u}{}'' \right)^2}{\sum_{j=1}^{N_j} \sum_{i=1}^{N_i} \left(\varphi_{ij,m}^{d}{}'' \right)^2} \tag{2.5}$$

where $\varphi_{ij,m}^{u}{}''$ and $\varphi_{ij,m}^{d}{}''$ are the values of the curvature of the m-th eigenmode at the point defined by the coordinates (i, j) of the tested structure in the undamaged (u) and in the damaged state (d).

The methods of using dynamic characteristics that can be helpful in locating damage in building structures are presented. The current interest in this topic and the method of recording and processing data is growing using the above and other indicators (compare (Żółtowski et al. 2023)).

3. EXPERIMENTAL RESEARCH

The aim of the experiment was to determine the location of the plate damage in two stages of destruction using selected methods presented above. Eigenvectors of the undamaged and damaged plate (two stages) were obtained in the experimental modal analysis by performing an impulse test. The literature provides several examples of studies conducted on two-dimensional objects; however, these differ from the present study in aspects such as boundary conditions, material properties (Navabian et al. 2016; Oyarzo Vera 2017; Sun et al. 2023; Zhong and Yang 2016).

3.1. Test object

The experimental object was a steel plate with dimensions of 85x100 cm and thickness of 1.5 mm, supported in two corners using elastic cords (Fig. 1).

The steel plate was prepared for testing by applying a grid of 110 measurement points forming 11 rows and 10 columns shown in Fig. 1. Then, the plate was hang up in the two upper corners using elastic cords, locating plate vertically. Accelerometers were attached at 8 selected grid points (28, 31, 43, 55, 69, 84, 97, 112) – sensors recording accelerations in the direction perpendicular to the plate plane. The sensors were positioned based on the premise that their placement should not coincide with a location of zero amplitude for eigenmodes. To select appropriate locations for the accelerometers, a free vibration eigenvalue analysis of a numerical model of the plate was conducted. The 356B18 PCB Piezotronics accelerometer model weighing 25 grams was used. The impulse test was carried out using a modal hammer (086C03 PCB Piezotronics) weighing 0.16 kg.

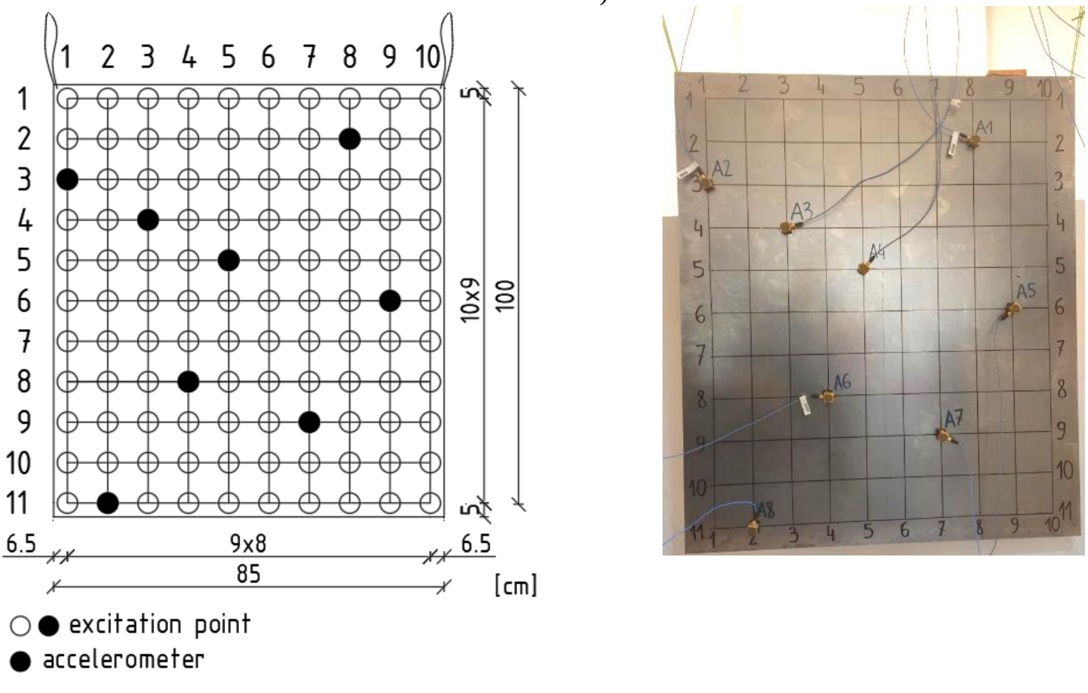

a) b)

Fig. 1. Steel plate: a) plate scheme: dimensions, point grid, sensor layout, b) experimental model of the plate with accelerometers

Three sets of tests (stages) were performed. The first on the undamaged plate (Scenario 0 – S0) and the next two on the damaged plate. The damage was made by cutting the plate diagonally in relation to its edge according to Fig. 2b) and 2c). In the second stage of the tests, the length of the damage was 16 cm (Scenario 1 – S1), and in the third stage 32 cm (Scenario 2 – S2). The damage location was chosen randomly; however, areas with zero vibration amplitude were excluded. Such a cut can represent mechanical damage to the plate or an area where corrosion has initiated and is propagating.

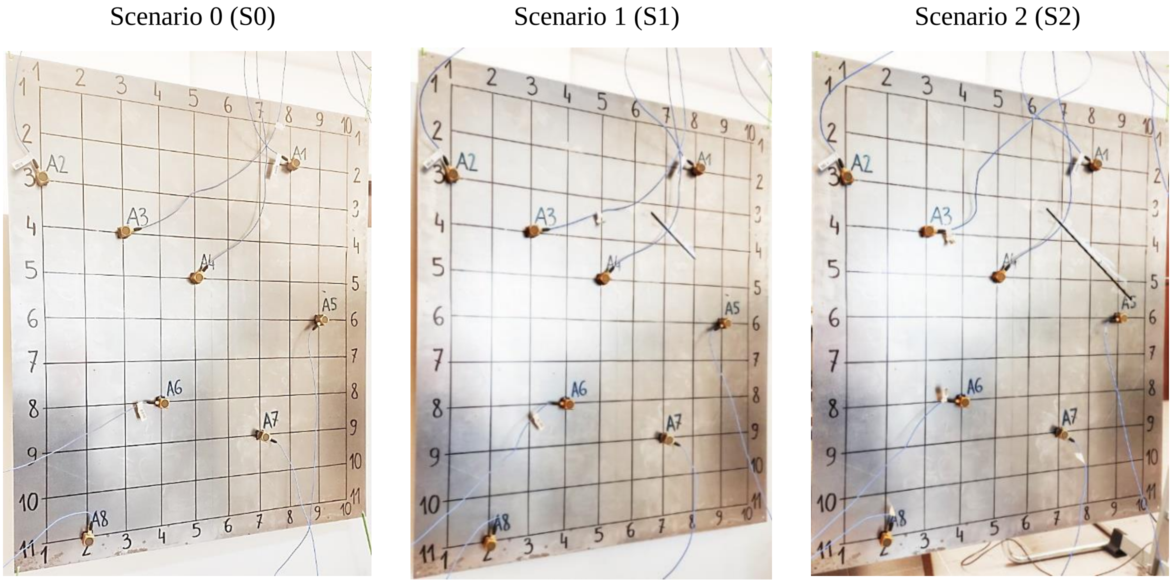

Scenario 0 (S0) Scenario 1 (S1) Scenario 2 (S2)

Fig. 2. Steel plate during three stages of testing: a) undamaged, b) damaged (damage length 16 cm), c) damaged (damage length 32 cm)

3.2. Measurement procedure and postprocessing

During each set of tests, a steel plate was excited with a modal hammer at each grid point. Each time, simultaneously the force and acceleration response of the plate were recorded at 8 selected points using accelerometers. A single test, performed as part of the test set for a given scenario, was conducted at a sampling rate of 3000 Hz, recording signals for approximately 20 s using the TEAC device. For each set of tests, the obtained signals were filtered in the MACEC program of the Matlab environment and then the modal characteristics were identified using the Poly-reference Least Squares Complex Frequency Domain (pLSCF) method available there. Modal characteristics were selected using the validation criteria – modal phase collinearity (MPC) with a value larger than 0.9, the stabilization criterium at the level of 1% for eigenfrequencies and 5% for damping ratios.

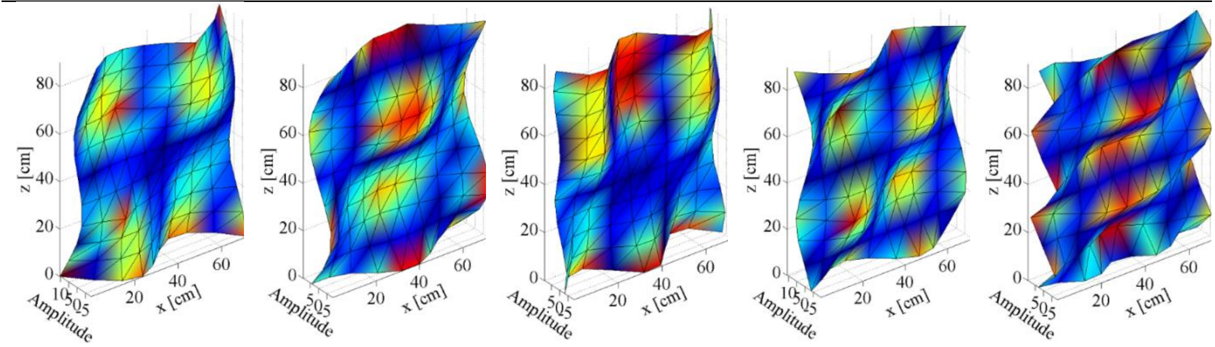
4. RESULTS AND DISCUSSION

As a result of the experimental tests, the excited eigenfrequencies of the plate (Tab. 1.) and the corresponding mode shapes (Fig. 3.) were obtained in three stages of the research: S0, S1 and S2.

Table 1. Eigenfrequencies of the steel plate and their changes due to damage in three scenarios

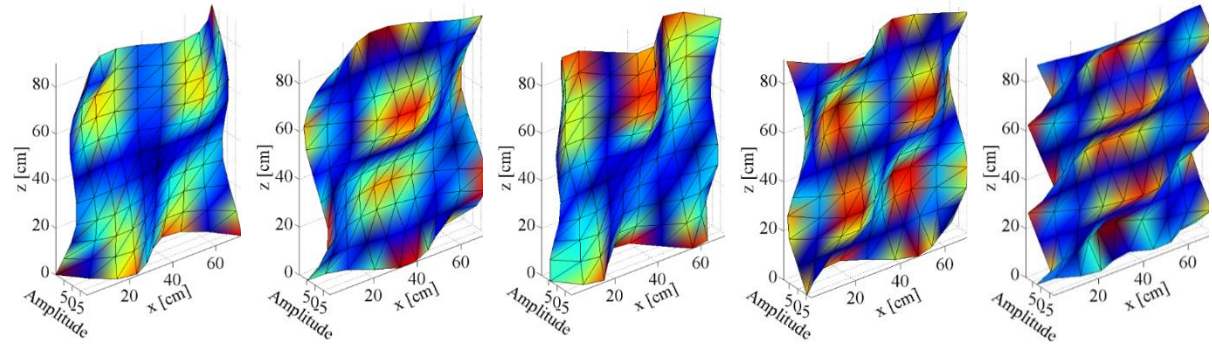
Mode	Eigenfrequencies *fi* [Hz]			Δ*fi* [%]		
	S0	**S1**	**S2**	**S0-S1**	**S0-S2**	**S1-S2**
1	36.64	36.58	36.30	0.16	0.93	0.77
2	44.75	44.72	44.36	0.07	0.87	0.81
3	64.59	64.39	63.75	0.31	1.30	0.99
4	68.18	68.02	66.77	0.23	2.07	1.84
5	136.06	135.41	135.41	0.48	0.48	0.00

Scenario 0 (S0)



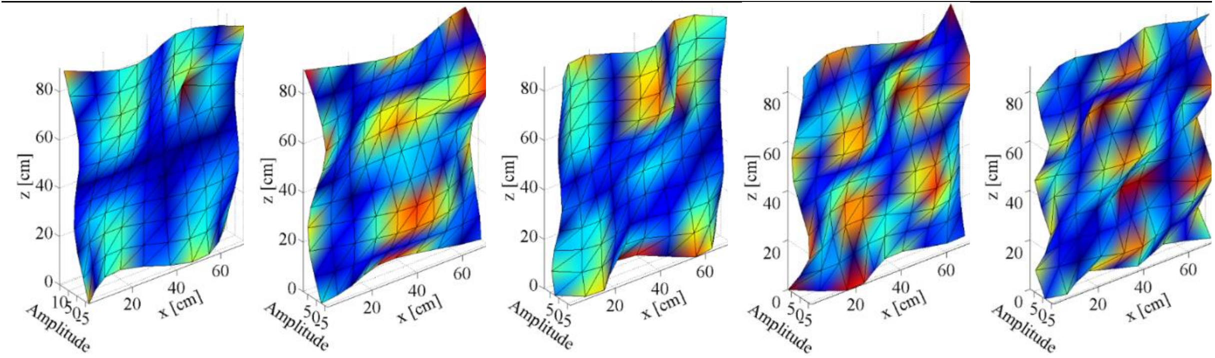
f_1=36.64 Hz f_2=44.75 Hz f_3=64.59 Hz f_4=68.18 Hz f_5=136.06 Hz

Scenario 1 (S1)



f_1=36.58 Hz f_2=44.72 Hz f_3=64.39 Hz f_4=68.02 Hz f_5=135.41 Hz

Scenario 2 (S2)

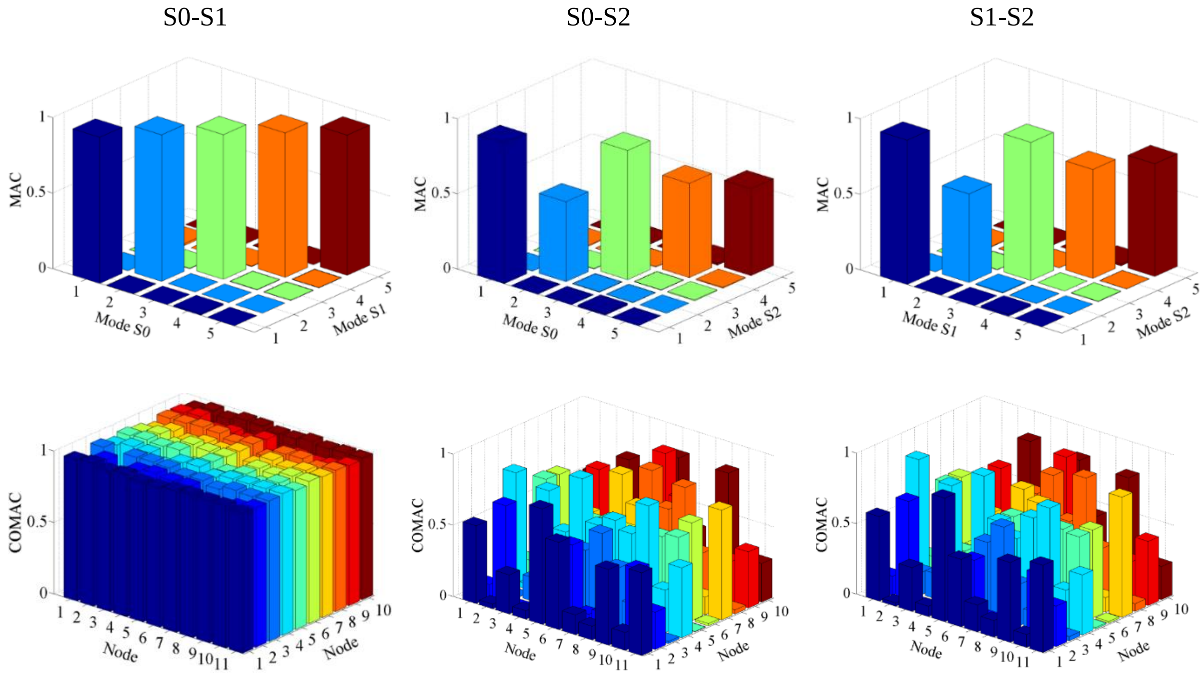


f_1=36.30 Hz f_2=44.36 Hz f_3=63.75 Hz f_4=66.77 Hz f_5=135.40 Hz

Fig. 3. Mode shapes and eigenfrequencies of a steel plate in three scenarios

Based on the obtained eigenvectors of the analyzed structure before and after failure, in the next step, the indicators of detection and location of damage were determined. Each time, the condition of the damaged plate from scenarios S1 and S2 was compared to the state of the undamaged plate S0 and the state of the damaged plate S2 to the state of S1.

First, the MAC and COMAC damage index were determined (Fig. 4.).

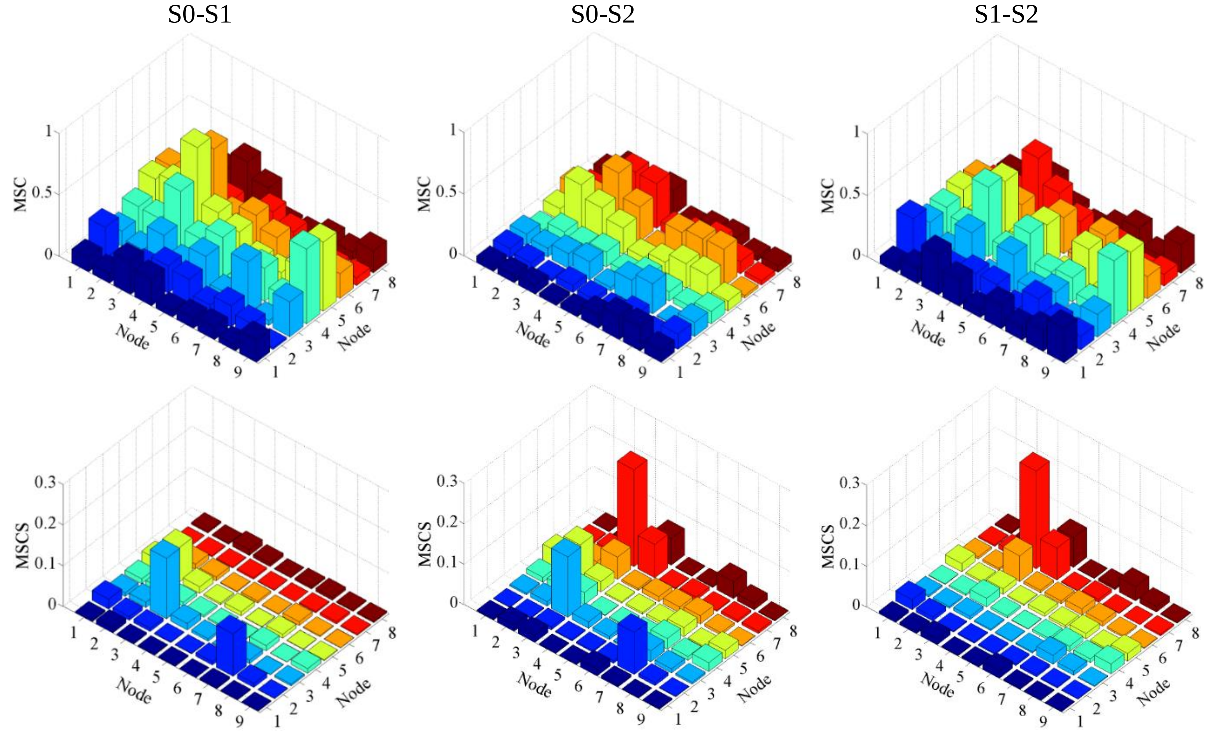


Rys. 4. MAC and COMAC indicators of steel plate (comparison of different scenarios)

By selecting the same – corresponding eigenmodes of the structure in two stages comparison of MAC and COMAC indicators were obtained. The results were presented using three-dimensional bar graphs: first for the plate in the undamaged state (S0) and in the damaged state (S1), and then in the states S0 and S2, S1 and S2 respectively (Fig. 4.). The MAC indices for different, non-corresponding eigenmodes are expected to be close to zero, which is confirmed by the top row of graphs in Fig. 4. However, when comparing eigenvectors of corresponding modes, for identical eigenvectors, the indices on the diagonal are 1.0. Due to the fact that the measurements are performed on real models that contain measurement noise, and the comparison concerns both undamaged and damaged structures or structures with different degrees of damage, the values of the indicators on the diagonal will not reach the value of 1.0. This is due to the fact that the compared vibration modes are not identical. The analysis of the MAC index for selected eigenmodes shows that when comparing the S0-S1 states, the indices are about 90%, while in the other two cases they are lower, in the range of 45%–85%. It can be stated that the modification from the S1 to S2 state change the eigenmodes more than the damage leading to the S1 state.

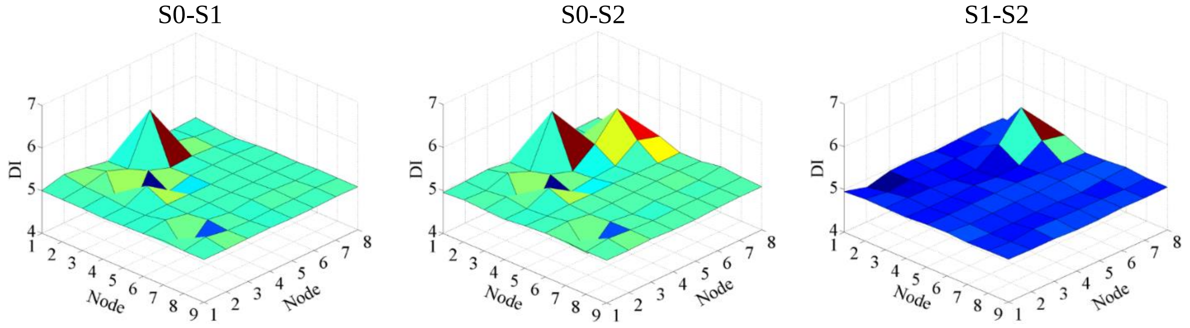
The lower row of graphs in Fig. 4. shows the COMAC indices, which allow for the localization of damage by comparing selected eigenmodes in the structure nodes for two different stages. Recalling that the value of 1 of the indicator refers to the complete convergence of the eigenmodes in the node, and the value of 0 to its absence, the obtained graphs do not provide clear information of damage localization in accordance with Fig. 2. Based on (Ewins 2000b), it was found that areas of the structure with low COMAC indices are also areas where the consequences of any discrepancies between two structural states are noticed, even when they are not directly next to each other. Therefore, the interpretation of COMAC indices can be problematic. The location of damage based on the obtained graphs may not be reliable, and the obtained results may only signal that damage may have occurred.

In the next step, the MSC and MSCS indices were determined based on a comparison of the curvatures of the eigenmodes of different stages of the plate. The results are presented in Fig. 5. In the graphs, the localization of damage of the structure will coincide with the higher bar. Comparing the obtained results of the MSC indices in the first and MSCS in the second row of Fig. 5. with the place of damage presented in Fig. 2., it can be stated that both indices contain information about the location of the real damage of the structure. The MSCS indices present this information much better, especially when comparing the S1 state with S2, where the graph clearly shows the location of the damage.


S0-S1 S0-S2 S1-S2

Rys. 5. MSC and MSCS indicators of steel plate (comparison of different scenarios)

The last one to be presented is the DI index (Fig. 6.), which is also based on the comparison of the curvatures of the eigenmodes. The obtained graphs present the damage localization of the steel plate in the clearest and most correct way among all the indicators discussed here.


S0-S1 S0-S2 S1-S2

Rys. 6. DI indicators of steel plate (comparison of different scenarios)

5. CONCLUSIONS

The research presented in this paper demonstrates the effectiveness of various vibration-based damage detection methods for locating damage in a steel plate as a example od two-dimensional structure. By comparing the dynamic characteristics of the structure in undamaged and damaged states and also damaged at various levels, it was possible to identify changes in modal parameters that signal the presence and location of damage. The most important remarks are noted below.

The Modal Assurance Criterion (MAC) and Coordinate Modal Assurance Criterion (COMAC) methods were able to detect changes in the structure between its undamaged and damaged states. However, while MAC provided clear overall comparisons of modal shapes, COMAC struggled with reliably pinpointing the exact location of damage. These methods are useful for damage detection, but their limitations in precise localization should be noted.

Both the Mode Shape Curvature (MSC) and Mode Shape Curvature Square (MSCS) indices proved to be more sensitive to structural changes and offered better localization capabilities compared to MAC and COMAC. In particular, the MSCS index demonstrated a higher accuracy in damage localization, especially when comparing stages of progressive damage, making it a more reliable tool for identifying damage places.

The Damage Index (DI), which also utilizes curvature data, provided the clearest and most accurate representation of the damage location.

The methods discussed can be effectively applied to real-world scenarios, especially in the context of SHM of critical infrastructures. The DI and MSCS indices, in particular, show promise for practical applications where accurate damage localization is necessary. However, care must be taken in selecting appropriate methods based on the complexity of the structure and the level of detail required for damage detection. Overall, the study underscores the importance of dynamic analysis techniques in non-destructive testing.

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