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Original Research Article

THE EFFECT OF THE WALL HEAT CAPACITY ON THE UNSTEADY TEMPERATURE DISTRIBUTION INSIDE BUILDINGS: A SIMPLE ANALYTICAL APPROACH

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Abstract

The best way to keep the required format of the manuscript is to overwrite these instructions with its text. Papers Heating buildings is important in everyday life. Especially today, every saving of thermal energy is important to stop the global warming of our planet. In this context, the influence of the thermal capacity of walls on the time-dependent temperature change inside buildings is most often ignored in the literature. Therefore, this work aims to investigate the influence of the thermal capacity of the wall on the time-dependent change of the internal temperature in a building room by developing a simple theoretical model enabling the calculation of unsteady heat transfer through the building wall, taking into account the role of the thermal capacity of the external wall. The theoretical analysis also takes into account the heat capacity of the air occurring in a limited cubic space, which has not been taken into account in other studies on this topic. Two cases of time-dependent changes in outdoor temperature are considered here: a constant outdoor temperature and a periodically changing ambient temperature. After applying a few simplifying assumptions, the problem can be reduced to a system of ordinary differential equations, which can then be solved analytically. Thus, the developed methodology can be used to design partitions in energy-efficient buildings.

Keywords: accumulation, unsteady temperature distribution, theoretical analysis, wall heat capacity

NOMENCLATURE:

a thermal diffusivity,$= k_s \,/\, ((\rho \bar{c}\,))$, m^2/s
$\bar{c}$ specific heat capacity of the wall, $J/(kg\ K)$

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c_p specific heat capacity of the air, $J/(kg\ K)$
A_w surface area of wall, m^2
h_{out} heat transfer coefficient in the ambient air outside the building, $W/(m^2K)$
h heat transfer coefficient inside the building, $W/(m^2K)$
H thickness of the wall, m
k_s thermal conductivity of the wall, $W/(m\ K)$
m mass of the wall, kg
m_a mass of the air, kg
T temperature, K
$\bar{T}$ average wall temperature, K
T_{out} ambient temperature, K
T_{in} internal temperature, K
T_p reference temperature, K
t time, s
Δt time lag, s
t_o half-period time, s
ρ density, kg/m^3
θ dimensionless temperature, $= T/T_p$
τ dimensionless time, $= ta/H^2$

1. INTRODUCTION

The imperative of global climate warming and the concerted efforts to mitigate its deleterious effects on the human environment have been the central focus of numerous scholarly endeavors. The analysis of the ramifications of temperature variations on the thermal dynamics of enclosed spaces assumes a paramount significance, as this issue stands as a pivotal factor impacting the occupants' well-being and everyday functionality. Correspondingly, the matter of operational expenditures, the reduction of which has been elevated to the zenith of priorities for the building owners, warrants close attention.

Consequently, an avid scholarly interest has been kindled in the domains of energy consumption optimization and the concomitant amelioration of its dissipation within inhabited spaces. This subject's significance is underscored by an expansive corpus of literature, wherein researchers underline the cardinality of ensuring room occupants' thermal comfort, intrinsically linked with the fulfillment of requisites pertaining to the energy characteristics of structures.

An example is shown in [1] in which the authors discussed the latest technologies and software that support the design and subsequent operation of modern buildings.

In works [2] and [3], the authors presented the properties of phase change materials (PCM) and solutions in various areas of their applications aimed at reducing energy consumption. In addition, they drew attention to the use of PCM as a thermal energy store (TES) from cooling processes produced by chilled water units. In turn, in [4], the authors, apart from appreciating the features and properties of phase change materials, pay attention to the negative effects of the use of these materials, such as weakening the mechanical properties or increasing the costs of producing these materials.

Due to the highest percentage of energy consumption generated by heat losses through partitions and the impact of thermal capacity on energy consumption and thermal comfort of users of residential and intended buildings as well as non-residential buildings, e.g. public buildings, the authors in [5] and [6] pay attention to the correct construction of these partitions. They also indicate the need to optimize projects that improve the thermal insulation of partitions and reduce heat losses.

In turn, in [7] the authors emphasize that various wall construction solutions, and in particular their thickness, have a strong impact on the time delay and the decrement coefficient. The materials from which these walls are made, having greater thermal inertia, allow for a lower loss coefficient. The obtained research results may be useful in designing appropriate external partitions for the building. Taking into account the important role of modeling in the issue of rational energy consumption, in [8] attention is drawn to models for assessing thermal comfort in terms of the energy performance of buildings. They further emphasize that optimal modeling reduces too high air temperature in the room, achieves the intended thermal state and minimizes energy consumption. The work [9] is devoted to modeling, in which the authors, using the method of simulation analysis of energy consumption, verified the compliance of the developed model by comparing it with the actual energy consumption. Based on the model, they analyzed the thermal design parameters of the building envelope structure and presented the optimal combination of energy saving effect schemes, which can provide decision support for energy saving in public buildings.

Like models, systems are also important in the process of rational energy management. In [10], the authors draw attention to the high popularity of systems designed to control temperature in rooms and present such a system. In the experiment they show that a room controlled using the developed system consumes much less cooling energy than a room controlled conventionally, and also shows fewer violations of comfort restrictions.

In this context, it is imperative to direct due diligence to the substantial consequences of the thermal capacitance inherent within building partitions or walls, vis-à-vis the efficacious harnessing of heat gain through accumulation. This focal point, elucidated across a plethora of investigations [11-15], has recently come under the purview of a scholarly group, whose published experimental inquiries [16-22] dissect the influence of building wall thermal capacitance, alongside the fluctuations in external temperatures, upon the indoor air temperature dynamics. These studies lay bare the oscillations and variations that take place within unheated spaces. Moreover, this empirical corpus attests to, and converges with, prior experimental and theoretical elucidations as documented in [23-31], which, inter alia, delve into aspects of the optimization of building wall insulation, the modulation of time lag and temperature attenuation by thermal insulation, and the dynamic behaviour of thermally-isolated walls.

However, the presented research is characterised by the absence of exhaustive theoretical analyses that expound upon the discussed phenomenon. Hence the need for theoretical inquiries, which in their essence, facilitate the methodical structuring of insights into the subject under investigation, and confer an elevated understanding of the essence of heat transfer through building walls, all the while circumventing formidable financial constraints.

Therefore, it is imperative to carry out theoretical investigations and corroborate them through empirical investigations particularly due to the dearth of theoretical works squarely addressing the intricacies of heat transfer in buildings. The present study presents an innovative approach to this conundrum by formulating a straightforward theoretical model.

An exceptionally intriguing theoretical contribution is the work [28], wherein the heat transfer through a building wall is analytically studied using Green's function, with climatic conditions congruent with those endemic to Northwestern Iran. Within the aforementioned study, damping and phase shifts of temperature distribution on both sides of the wall were delineated contingent upon variances in external conditions, the material composition of the wall, and the heat transfer coefficient. However, the reported work has a hidden limitation in that it does not take into account the heat capacity of the air confined in the internal spaces of the building's structural system.The theoretical model developed in our current work encompasses the internal air capacitance and also defines damping and temperature distribution shifts for various wall parameters and external conditions.

The objective of this study is, therefore, a simplified analytical analysis of the temporal temperature profile within an enclosed space, considering walls with varying thermal capacities that enclose the examined area under changing external temperature conditions. Two distinct scenarios have been subjected to analysis:
1 – non-stationary conditions of cyclic wall and interior heating, and
2 – non-stationary wall heating under constant external temperatures.

The motivation for the simplified analytical approach are standard solutions to heat flow problems along with a discussion of the most important boundary value problems and theories regarding the Green's function method for calculating the unsteady heat flow through the building wall, presented e.g. in [32, 33].

Based on the presented literature review, it can be concluded that our work took into account the influence of air located in a limited internal space on temperature damping and phase shift. Therefore, this work brings new value to the current state of scientific and technical knowledge and stands out from other studies that did not take into account the influence of the air in the internal space on the temperature damping and phase shift in the presented issues.

2. MATHEMATICAL MODEL

In the following we are concerned with the problem sketched in Fig. 1. Here an unsteady heat flux q is transferred from the external air (ambient) with the temperature T_{out} to the enclosure in the building with an internal air temperature T_{in} ($T_{out} > T_{in}$), through the external wall with a thickness H, and an area A_w. The wall is assumed to have an average temperature $\bar{T}$ and the heat capacity $m\,\bar{c}$.

The internal air has the heat capacity $m_p c_p$. The heat diffusivity for the wall material is $a = k/(\rho\, c)$. In the considered theoretical model, it was assumed that the considered enclosure is surrounded by an insulation, and the specific heat q can only flow through one wall with the area A_w. (see Fig. 1). Then, when the temperature inside the space exceeds the outside temperature $T_{out} < T_{in}$, the direction of the heat flow changes to the opposite direction. This process can be repeated periodically for a periodic temperature variation in the ambient or monotonically if the ambient temperature outside the building is constant.

The theoretical model assumes that the temperature in the wall is uniform at all times.
The heat transfer coefficients on both sides of the wall are equal to: h_{out} - from the ambient air to the outside surface of the wall and h - from the inside wall surface to the inside air.
Sample parameters of the walls and the indoor space are listed in table 1.

Table 1. Parameters of the wall and the indoor space

Wall	H, m	$m\,\bar{c},\, MJ/K$	$A_w,\, m^2$	$a,\, m^2/s$	$m_a\, c_p, MJ/K$	$h_{out}, W/(m^2K$	$h, W/(m^2K)$
B1	0.458	3.466	14	$1.22 \cdot 10^{-7}$	0.111	10	10
B2	0.393	1.170	14	$1.93 \cdot 10^{-7}$	0.111	10	10

The heat balance for the wall and the internal air space shown in Figure 1 is described by equations 2.1 and 2.2 (for $H^2/A_w \ll 1$ and the air is assumed to be well mixed due to the presence of free convection inside the room):

- heat balance for the external wall

$$\frac{d}{dt}\left(m\,\bar{c}\,\bar{T}(t)\right) = -hA_w(\bar{T} - T_{in}) + h_{out}A_w(T_{out} - \bar{T}), \tag{2.1}$$

- heat balance for the indoor space

$$\frac{d}{dt}\left(m_a\, c_p\, T_{in}(t)\right) = hA_w(\bar{T} - T_{in}), \tag{2.2}$$

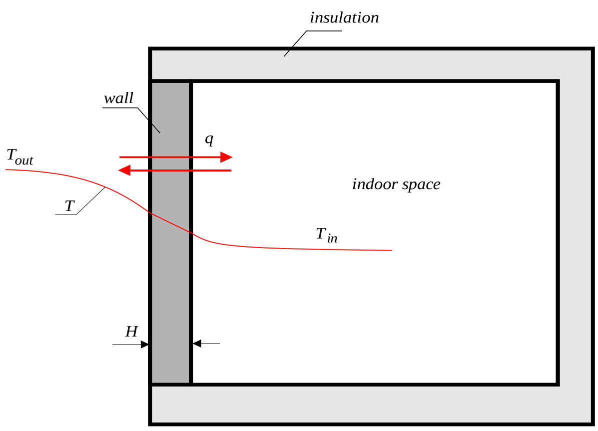

Fig. 1. Heat inflow or outflow from the internal space, temperature distribution

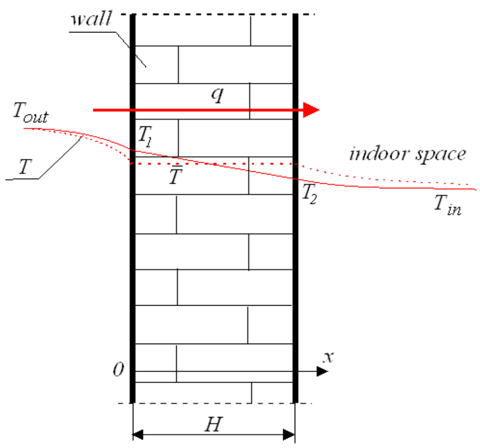

Fig. 2. Building wall, temperature distribution at any time

Quasi-stationary heat conduction through the building wall at a specified time t is shown in Fig. 2. The transferred heat by convection on the outer wall surface of the building is can be expressed by (see Figs. 1, 2):

$$q = h_{out}(T_{out} - T_1) \tag{2.3}$$

Evaluating the conductive and convective heat fluxes for the wall at any time results in

$$\frac{k_s}{H}(T_1 - T_2) = h_{out}(T_{out} - T_1), \tag{2.4}$$

where the temperatures T_1 and T_2 are the surface temperatures of the outer surface and inner surface of the wall, respectively. Now we assume for simplification an average temperature in the wall which is equal to

$$\bar{T} = \frac{\int_0^H T(x)dx}{H} = \frac{T_1+T_2}{2} \to T_2 = 2\bar{T} - T_1, \tag{2.5}$$

Using equations (2.4) and (2.5) the temperature at the wall surface T_1 as a function of the average temperature is given by the equation:

$$T_1 = \frac{2k_s}{2k_s+h_{out}H}\,\bar{T} + \frac{h_{out}H}{2k_s+h_{out}H}T_{out}. \tag{2.6}$$

The convective heat flux on the outer wall depending on the average wall temperature $\bar{T}$ is now equal to:

$$h_{out}(T_{out} - T_1) = \frac{h_{out}}{1+\frac{h_{out}H}{2k_S}}(T_{out} - \bar{T}) = h'_{out}(T_{out} - \bar{T}) \tag{2.7}$$

Please note that if the thermal conductivity of the wall is large, $k_s \to \infty$, or (and) the thickness is small, $H \to 0$, then the heat transfer coefficients on the outer surface of the wall are equal, $h'_{out} = h_{out}$. A similar situation is on the inner surface of the wall. Building walls are generally poor heat conductors, therefore the heat transfer coefficients should be reduced accordingly in the equations (2.1 - 2.2).

Under real conditions, the outside air temperature fluctuates in the summer. At night, the outside air temperature is lower than during day time, therefore it is reasonable to model the variation of the temperature with time by a periodic function. Thus, two cases of unsteady heat transfer from the outside to the inside space are considered in the following:
1 – A sinusoidal change of the outside temperature and
2 – A constant outside temperature: a high temperature for the case of heating of the room and a low temperature in case of cooling of the room.

2.1. Unsteady, periodic heating of the outside wall and the inner space

It will be assumed that the outside ambient air temperature distribution over time is given by

$$T_{out} = T_{outO} + T_{outA}\, sin\left(\frac{\pi}{t_0} t\right), \tag{2.8}$$

where T_{out0} is the average outside temperature of the ambient air, T_{outA} is the outside air temperature amplitude, t_0 is the half-period of the outside temperature change.

By introducing the following dimensionless quantities

$$\theta = \frac{\bar{T}}{T_p};\ \theta_{out} = \frac{T_{out}}{T_p};\ \theta_{aut0} = \frac{T_{auto}}{T_p};\ \theta_{outA} = \frac{T_{outA}}{T_p};\ \theta_{in} = \frac{T_{in}}{T_p};\ \tau = \frac{t\,a}{H^2}. \tag{2.9}$$

into the heat balance equations (2.1, 2.2), a dimensionless system of ordinary differential equations can be obtained:

$$\frac{d\theta}{d\tau} + A\,\theta = (A - B)\,\theta_{in} + B\,\theta_Z\ ; \qquad \frac{d\theta_{in}}{d\tau} = C(\theta - \theta_{in}), \tag{2.10}$$

where:

$$A = \frac{hF_wH^2}{a\,m\,\bar{c}};\ B = \frac{h_{out}F_a\,H^2}{a\,m\,\bar{c}};\ C = \frac{hF_aH^2}{a\,m_a\,c_p}. \tag{2.11}$$

The energy equations (2.10) form a system of ordinary differential equations for the two unknown temperatures θ and θ_{in}. After transformations and additional differentiation, equation (2.9) leads to a inhomogeneous linear ordinary differential equation of the second order:

$$\frac{d^2\theta_{in}}{d\tau^2} + \alpha\ \frac{d\theta_{in}}{d\tau} + \beta\ \theta_{in} = \beta\ \theta_{out}, \tag{2.12}$$

where:

$$\alpha = A + B + C;\ \beta = B \cdot C, \tag{2.13}$$

and the assumed external temperature distribution in time is described by the equation:

$$\theta_{out} = \theta_{Oout} + \theta_{OA} sin\left(\frac{\pi}{\tau_0}\tau\right). \tag{2.14}$$

The differential equation (2.9) satisfies the initial conditions for the indoor temperature

$$\theta_{in}(0) = 1; \qquad \frac{d\theta_{in}}{d\tau}(0) = 0. \tag{2.15}$$

Using the solution of equation (2.4), the wall temperature is then calculated using the equation

$$\theta = \theta_{in} + \frac{1}{C}\frac{d\theta_{in}}{dt} \tag{2.16}$$

The procedure for solving equation (2.12) is as follows:

The solution of the homogeneous differential equation

$$\frac{d^2\theta_{in}}{d\tau^2} + \alpha\ \frac{d\theta_{in}}{d\tau} + \beta\ \theta_{in} = 0, \tag{2.17}$$

depends on the value of the discriminant of the characteristic equation

$$\Delta = \alpha^2 - 4 \cdot 1 \cdot \beta, \tag{2.18}$$

For the case when $\Delta > 0$, the roots of the characteristic equation are given by

$$r_{1,2} = \frac{-\alpha \pm \sqrt{\Delta}}{2} = -\frac{\alpha}{2} \pm \frac{1}{2}\sqrt{\Delta}, \tag{2.19}$$

and the general solution to the homogeneous differential equation is

$$\theta_{inh} = C_1 \cdot e^{r_1 \cdot \tau} + C_2 \cdot e^{r_2 \cdot \tau}. \tag{2.20}$$

The particular solution is provided in the form of:

$$\theta_{inpar} = E + F \cdot sin\left(\frac{\pi}{\tau_0}\tau\right) + G \cdot cos\left(\frac{\pi}{\tau_0}\tau\right). \tag{2.21}$$

In the here considered case, the right hand side of equation (10) is equal to:

$$\beta\theta_{out} = \beta\theta_{out0} + \beta\theta_{outA} sin\left(\frac{\pi}{\tau_0}\tau\right). \tag{2.22}$$

After substituting the special solution into equation (2.4), we obtain an equation in which there are unknown constants E, F and G

$$-F\cdot\left(\frac{\pi}{\tau_0}\right)^2 sin\left(\frac{\pi}{\tau_0}\tau\right)-G\cdot\left(\frac{\pi}{\tau_0}\right)^2 cos\left(\frac{\pi}{\tau_0}\tau\right)+\alpha\cdot F\cdot\left(\frac{\pi}{\tau_0}\right)cos\left(\frac{\pi}{\tau_0}\tau\right)$$

$$-\alpha\cdot G\cdot\left(\frac{\pi}{\tau_0}\right)sin\left(\frac{\pi}{\tau_0}\tau\right)+\beta\cdot E+\beta\cdot F\cdot sin\left(\frac{\pi}{\tau_0}\tau\right)+\beta\cdot G\cdot cos\left(\frac{\pi}{\tau_0}\tau\right)$$

$$=\beta\theta_{out0}+\beta\theta_{outA}sin\left(\frac{\pi}{\tau_0}\tau\right). \quad (2.23)$$

Comparing the coefficients for the sine and cosine functions separately, we obtain a system of three algebraically equations for the constants E, F, G:

$$\beta E=\beta\theta_{out0},$$

$$-F\cdot\left(\frac{\pi}{\tau_0}\right)^2-\alpha\cdot G\cdot\left(\frac{\pi}{\tau_0}\right)+\beta\cdot F=\beta\,\theta_{ZA},$$

$$-G\cdot\left(\frac{\pi}{\tau_0}\right)^2+\alpha\cdot F\cdot\left(\frac{\pi}{\tau_0}\right)+\beta\cdot G=0. \quad (2.24)$$

The constants E, F and G obtained from solving the above system of equations are given by

$$E=\theta_{out0};\ F=-\frac{\beta\left[\left(\frac{\pi}{\tau_0}\right)^2-\beta\right]\theta_{outA}}{\left[\left(\frac{\pi}{\tau_0}\right)^2-\beta\right]^2+\alpha^2\left(\frac{\pi}{\tau_0}\right)^2}\ \text{i}\ G=-\frac{\alpha\beta\left(\frac{\pi}{\tau_0}\right)\theta_{outA}}{\left[\left(\frac{\pi}{\tau_0}\right)^2-\beta\right]^2+\alpha^2\left(\frac{\pi}{\tau_0}\right)^2}. \quad (2.25)$$

The values of exemplary constants appearing in the above equations are included in table 2 for $\theta_{out0}=1$; $\theta_{outA}=0.5$

Table 2. Exemplary constants for the solution of the problem

	A	B	C	τ_0	α	β	E	F	G
B1	71.5	71.5	2 308	0.025	2 451	165 000	1	0.103	- 0.216
B2	109	109	972	0.054	1 190	106 000	1	0.382	- 0.223

The temperature field inside the room describes the solution of the inhomogeneous equation (2.4) being the sum of the homogeneous solution and a particular solution:

$$\theta_W = E + C_1 e^{r_1\tau} + C_2 e^{r_2\tau} + F sin\left(\frac{\pi}{\tau_0}\tau\right) + G cos\left(\frac{\pi}{\tau_0}\tau\right), \tag{2.26}$$

$$\frac{d\theta_{in}}{d\tau} = C_1\, r_1\, e^{r_1\tau} + C_2\, r_2\, e^{r_2\tau} + \frac{\pi}{\tau_0} F\, cos\left(\frac{\pi}{\tau_0}\tau\right) - \frac{\pi}{\tau_0} G\, sin\left(\frac{\pi}{\tau_0}\tau\right), \tag{2.27}$$

The average wall temperature is described by the equation:

$$\theta = \theta_{in} + \frac{1}{C}\frac{d\theta_{in}}{d\tau} = E + C_1\, e^{r_1\tau} + C_2\, e^{r_2\tau} + F\, sin\left(\frac{\pi}{\tau_0}\tau\right) + G\, cos\left(\frac{\pi}{\tau_0}\tau\right)$$

$$+\frac{1}{C}\left(C_1\, r_1\, e^{r_1\tau} + C_2\, r_2\, e^{r_2\tau} + \frac{\pi}{\tau_0}\, F\, cos\left(\frac{\pi}{\tau_0}\tau\right) - \frac{\pi}{\tau_0}\, G\, sin\left(\frac{\pi}{\tau_0}\tau\right)\right), \tag{2.28}$$

using the initial conditions

$$\theta_{in}(0) = 1 \text{ and } \frac{d\theta_{in}}{d\tau}(0) = 0, \tag{2.29}$$

The integration constants can be calculated:

$$C_1 = -(1 - E - G)\frac{r_2}{r_1 - r_2} - F\left(\frac{\pi}{\tau_0}\right)\frac{1}{r_1 - r_2};$$

$$C_2 = (1 - E - G)\frac{r_1}{r_1 - r_2} + F\left(\frac{\pi}{\tau_0}\right)\frac{1}{r_1 - r_2}. \tag{2.30}$$

For the two examples specified above, the numerical values of the parameters in equations (2.12 – 2.15) and the above constants are equal to:

for wall $B1$: $r_1 = -69.5;\ r_2 = -2382;\ C_1 = 0.216;\ C_2 = -0.00082,$
for wall $B2$: $r_1 = -96;\quad r_2 = -1095;\ C_1 = 0.225;\ C_2 = -0.00200.$

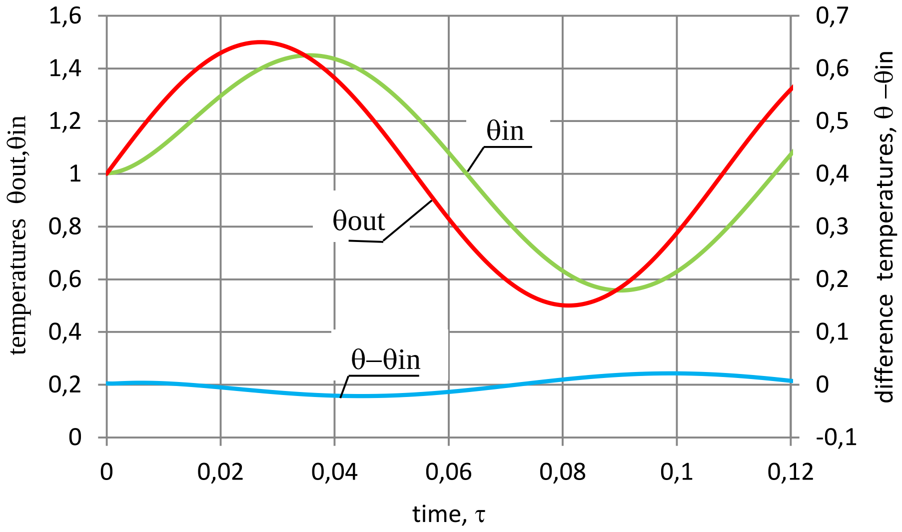

Fig. 3. Dimensionless temperature distributions for the lightweight wall B2

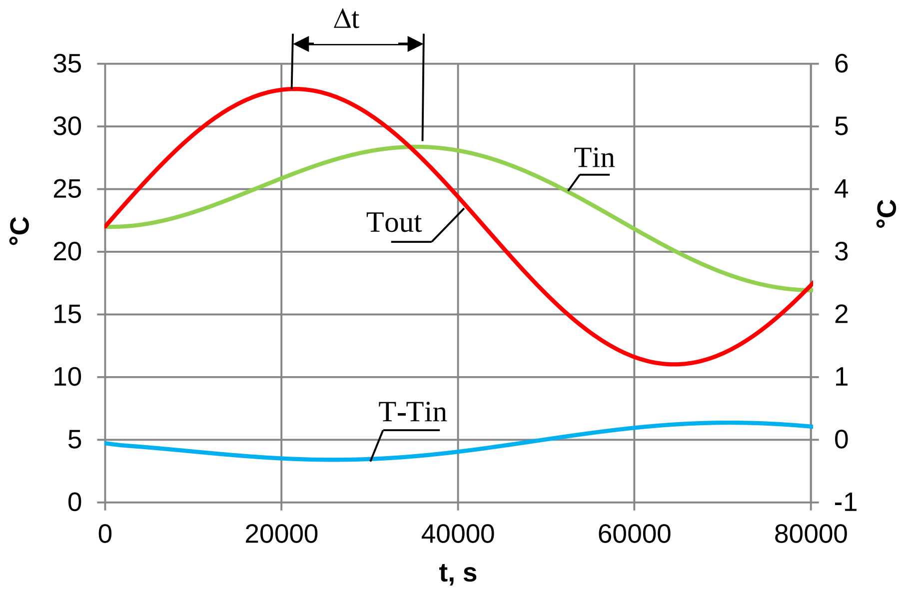

Fig. 4. Dimension temperature distributions for the heavy wall *B1* ($\Delta t = 3.9\ h;\ \Delta T_{max} = 4$°C)

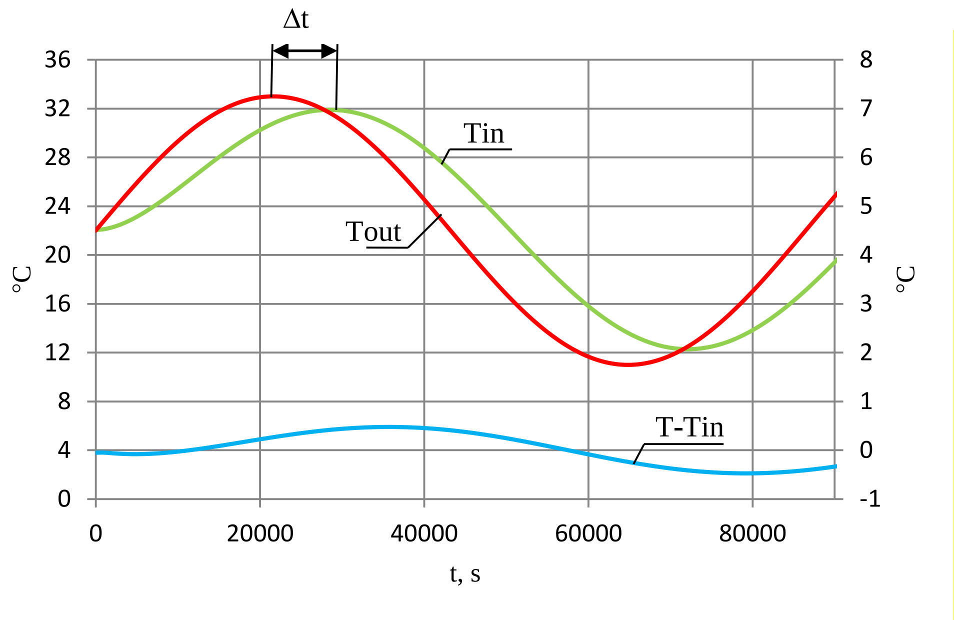

Fig. 5. Dimension temperature distributions for the lightweight wall *B2* ($\Delta t = 2.5\ h;\ \Delta T_{max} = 2.5K$)

The results of the theoretical study is presented in Figures 3 - 5, which show that the decrease in internal temperature ΔT_{max} and the time lag Δt compared to the external temperature depend on the heat capacity of the wall. The indoor temperature T_{in} changes periodically, similar to the outdoor ambient temperature T_{out}. The direction of the heat flow in the wall q changes depending on the sign of the wall temperature difference and the temperature inside the room $T - T_{in}$ (see Fig. 1). For a heavy wall (*B1*) the temperature damping and delay parameters are greater than for a light wall (*B2*). Interesting is the temperature difference, $T - T_{in}$, between the temperature of the wall and the temperature inside the space, which is very small.

2.2. Unsteady heating and cooling of the room with a constant outside air temperature

The heat flow from the outside to the room inside is described by equations (2.4):

$$\frac{d^2\theta_{in}}{d\tau^2} + \alpha\,\frac{d\theta_{in}}{d\tau} + \beta\,\theta_{in} = \beta\,\theta_{out}\ ; \qquad \theta = \theta_{in} + \frac{1}{C}\frac{d\theta_{in}}{dt}. \tag{2.31}$$

Assuming a constant outside air temperature in the first differential equation $\theta_{out} = const$ and substituting the expression

$$y(\tau) = \beta(\theta_{in}(\tau) - \theta_{out}) \tag{2.32}$$

The following differential equation can be obtained

$$\frac{d^2y}{d\tau^2} + \alpha\,\frac{dy}{d\tau} + \beta\,y = 0, \tag{2.33}$$

whose solution is given by

$$y(\tau) = C_1 e^{r_1 \tau} + C_2 e^{r_2 \tau}. \tag{2.34}$$

The parameters r_1 and r_2 are the roots of the characteristic equation, and the integration constants C_1 and C_2 are determined from the initial conditions:

$$y(0) = \beta(1 - \theta_{out})\,; \qquad \frac{dy}{d\tau}(0) = 0. \tag{2.35}$$

After substituting the calculated integration constants into the solution (2.27), the temperature inside the space and the derivative of the inside temperature are described by the equations

$$\theta_{in} = \theta_{out} + (1 - \theta_{out})\frac{r_2 e^{r_1 \tau} - r_1 e^{r_2 \tau}}{r_2 - r_1}, \quad \frac{d\theta_{in}}{d\tau} = (1 - \theta_{out})\frac{r_1 r_2}{r_2 - r_1}\left(e^{r_1 \tau} - e^{r_2 \tau}\right). \tag{2.36}$$

3. RESULTS METODOLOGY AND DISCUSSION

The theoretical method presented in this work is simpler than existing works and can be used in the design of building partitions ensuring appropriate thermal comfort of rooms. Modeling of the structure of these partitions, as well as their optimization, contributes to obtain the most advantageous solution in the arrangement of partition layers in terms of saving thermal energy and minimizing heat losses.

The time dependent variation of the temperature in the internal room, which is shown in Fig. 6 for a constant external temperature, changes exponentially over time.

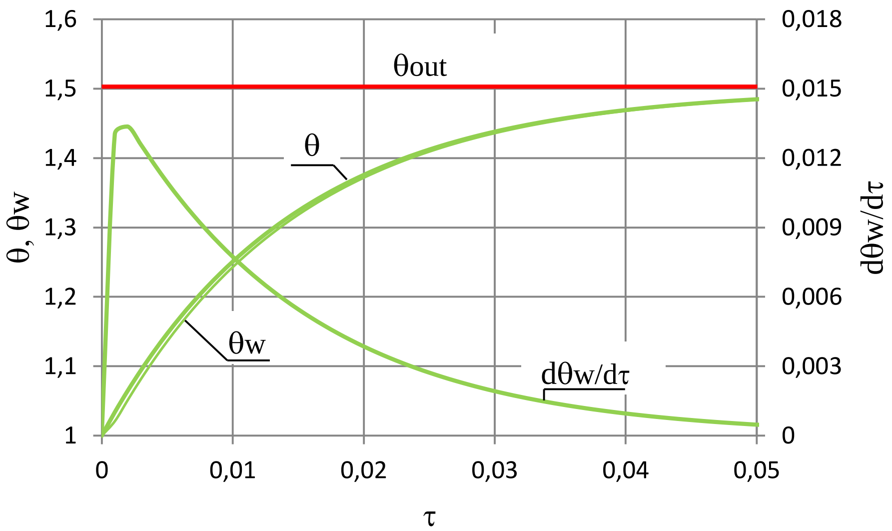

Fig. 6. Distribution of the internal temperature and its derivative

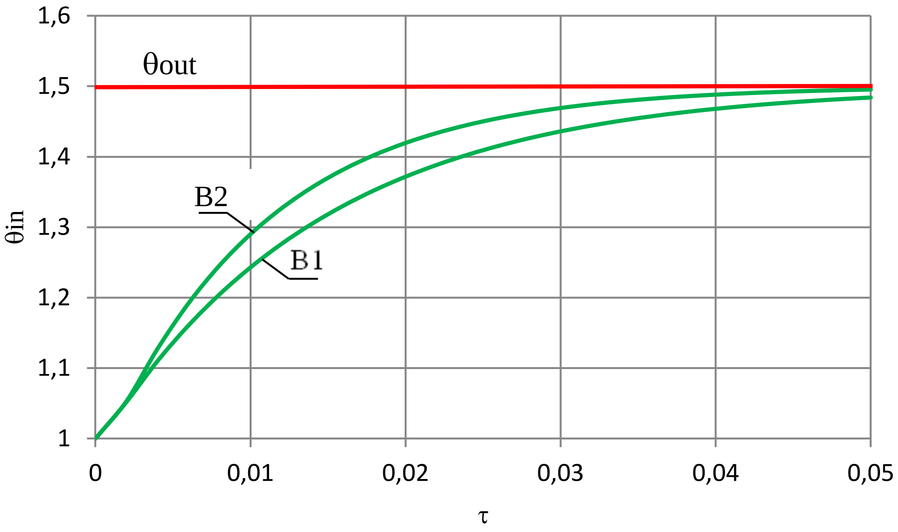

Fig. 7. Dimensionless temperature distributions in the heated space ($\theta_{out} = 1.5$)

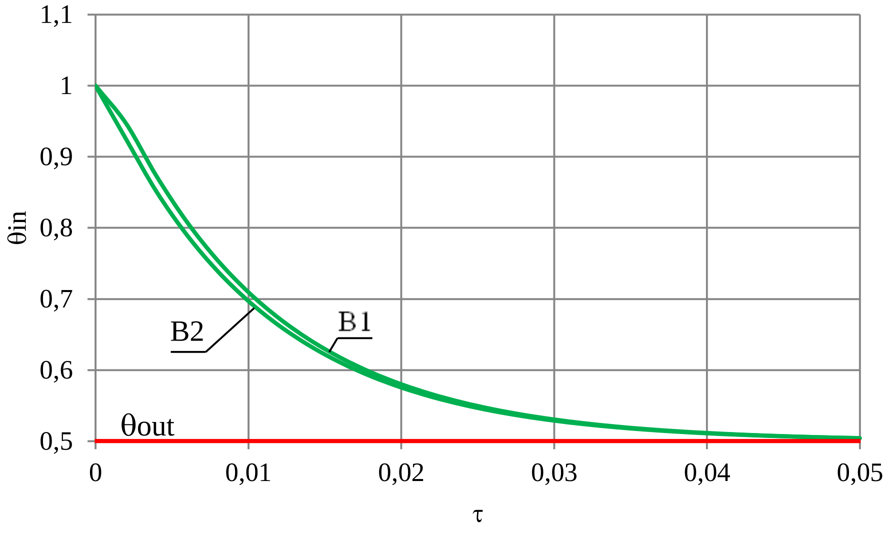

Fig. 8. Dimensionless temperature distributions in the cooled space ($\theta_{out} = 0.5$)

Figures 7 - 8 show the dependence of the temperature inside the room for two different wall heat capacities at constant outside air temperatures. For a heavy wall (*B1*) the temperature changing less than for a light wall (*B2*).

A mixed research methodology was used to solve the research problem based on a combination of a qualitative approach regarding understanding and interpretation of the phenomenon of external wall heat capacity and a quantitative approach describing this phenomenon using numerical data.
The mixed research methodology used allowed for a better understanding of the phenomenon of wall thermal capacity and has a significant impact on the research results, which, in accordance with the best practices for presenting quantitative data, include the use of appropriate charts.

Compared for the two cases considered, namely a wall with a low thermal capacity (B1) and a wall with a high thermal capacity (B2), they constitute the basis for an individual analysis and evaluation of the results. It also provides an opportunity to compare the achievements of other researchers who, in their studies on this subject, did not take into account the heat capacity of air occurring in a limited space.

Due to its universality of use, the method can be practically useful in simplified energy analyzes of buildings, which use the temperature values of the building envelope and inside a given room, as well as the value of the heat capacity of the air present in a limited cubic space. In such cases, the results obtained can be used to assess the energy efficiency of a facility and planning the costs of possible thermal modernization projects in order to meet the normative technical requirements for appropriate thermal insulation of building envelopes.

Moreover, the thermal capacity of given values assigned to a specific building partition is related to the degree of effective use of heat gains, which is taken into account in the usable energy balance in the case of heating and ventilation. This is then appropriately used in the economic assessment of the operating costs of a given building.

4. CONCLUSIONS

Based on the results of the theoretical research presented in this work, it can be concluded that parameters such as the decrease in internal temperature and the delay in relation to the external temperature depend on the thermal capacity of the wall. The greater the thermal capacity of the wall, the greater are the mentioned changes, and the obtained results are consistent with theoretical and experimental studies cited in the scientific literature.

In addition, the benefit of this work is the ability to determine based on the obtained temperature damping value, the degree of the maximum daily fluctuation of the external temperature, and the maximum daily fluctuation of the temperature of the internal surface of the building envelope.

A valuable advantage of this work is that the theoretical model created is universal and can be used for various external temperature distributions. This was confirmed by the example external conditions tested here: cyclic, see equation 10, and constant temperature, see equation 2.18.
In future research, it is planned to use the developed theoretical model for cases involving different external temperature distributions, apart from those presented in this Article. An interesting option that deserves research interest in the future is to adopt theoretical parameters for several representative partitions from the groups of massive, reinforced and light partitions, and then verify the theory and the obtained calculation results through a research experiment.

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